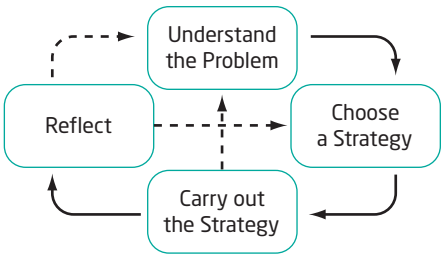
9.1 Investigate Measurement Concepts

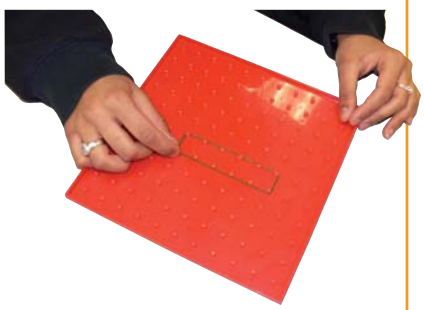
This chapter includes several investigations involving measurement concepts you studied in Chapter 8. When conducting an investigation, use the problem solving process introduced in Chapter 1.



Investigate A

Tools

- geoboard
- elastic bands



How can you model the areas of rectangles with the same perimeter?

You have 12 m of rope to fence off a rectangular play area at a summer day camp.

Method 1: Use a Geoboard

Use a geoboard to explore the different rectangles that can be formed with a perimeter of 12 m.

1. Let the distance between the pegs on the geoboard represent 1 m.

a) Use an elastic band to construct a rectangle that represents the play area.

b) What are the dimensions of the rectangle? Calculate the area. Use a table like this one to record your results.

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1			12	
2			12	
3			12	

c) Construct additional rectangles with the same perimeter and record your results. How many different rectangles were you able to create?

2. a) Which rectangle had the least area? What are its dimensions? Describe its shape.

b) Which rectangle had the greatest area? What are its dimensions? Describe its shape.

3. **Reflect** Which shape would you choose for the play area at the day camp? Give reasons for your choice.

Method 2: Use *The Geometer's Sketchpad*®

Use *The Geometer's Sketchpad*® to investigate the areas of rectangles with a fixed perimeter of 12 m.

Tools

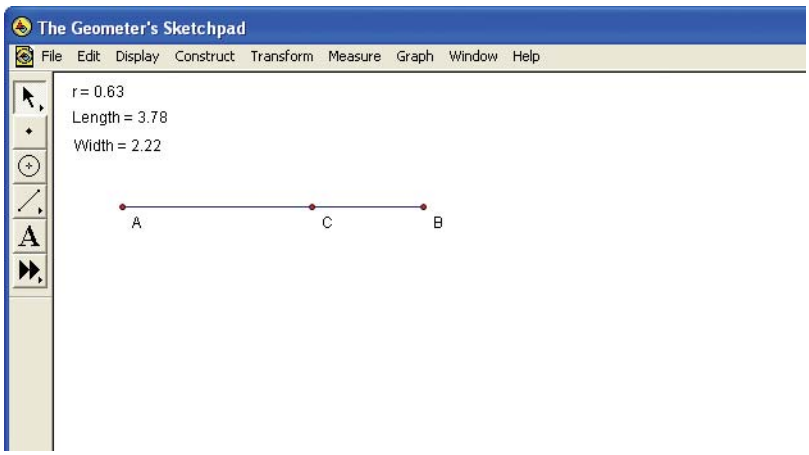
- *The Geometer's Sketchpad*® software
- computers

1. From the **Edit** menu, choose **Preferences**. Click on the **Text** tab. Ensure that **For All New Points** is checked. Click on **OK**.

2. Construct a slider to control the dimensions of the rectangle.
   - Construct a horizontal line segment AB.
   - Construct a point C on the line between A and B.
   - Select points A, B, and C, in that order. From the **Measure** menu, choose **Ratio**. The ratio AC:AB will appear on the screen.
   - To change the label of AC:AB, select this ratio measurement. Right click and choose **Label Measurement** from the drop-down menu. Change the label to **r**.
   - Drag the point C back and forth. Note how the ratio **r** changes. This forms a slider that can be used to control the dimensions of a rectangle.

3. Select the ratio **r**. From the **Measure** menu, choose **Calculate**. Enter the formula 6*__r__ by selecting **r** from the **Values** drop-down menu on the calculator. Change the label to **Length**.

4. Select **r** again. From the **Measure** menu, choose **Calculate**. Enter the formula 6*(1 − **r**). Change the label to **Width**.



5. Construct a point D in the workspace. Select point D. From the **Measure** menu, choose **Abscissa (x)**. Select point D again. Then, from the **Measure** menu, choose **Ordinate (y)**. These are the coordinates of point D.

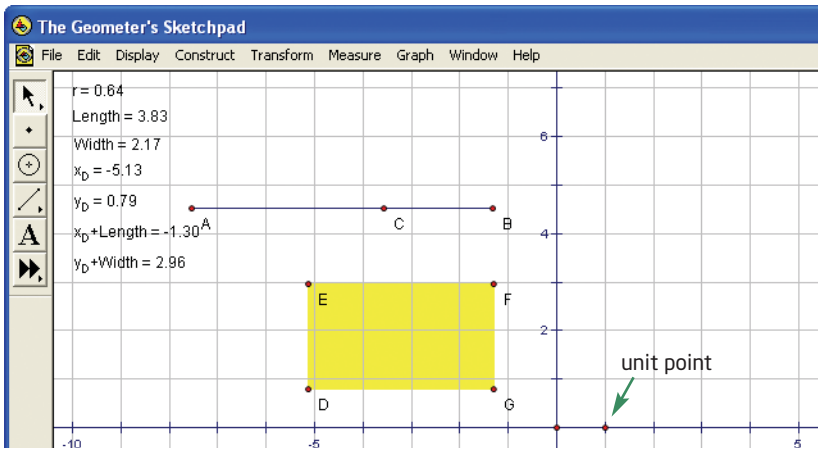
6. Select $\mathbf{x_D}$ and **Length**. From the **Measure** menu, choose **Calculate**. Enter the formula $\mathbf{x_D}$ + **Length**.

7. Select $\mathbf{y_D}$ and **Width**. From the **Measure** menu, choose **Calculate**. Enter the formula $\mathbf{y_D}$ + **Width**.

8. Plot the remaining three points to form the vertices of a rectangle DEFG.
   - Select $\mathbf{x_D}$ and $\mathbf{y_D}$ **+ Width**, in that order. From the **Graph** menu, choose **Plot As (x, y)**. This is point E.
   - Select $\mathbf{x_D}$ **+ Length** and $\mathbf{y_D}$ **+ Width**. From the **Graph** menu, choose **Plot As (x, y)**. This is point F.
   - Finally, select $\mathbf{x_D}$ **+ Length** and $\mathbf{y_D}$. From the **Graph** menu, choose **Plot As (x, y)**. This is point G.

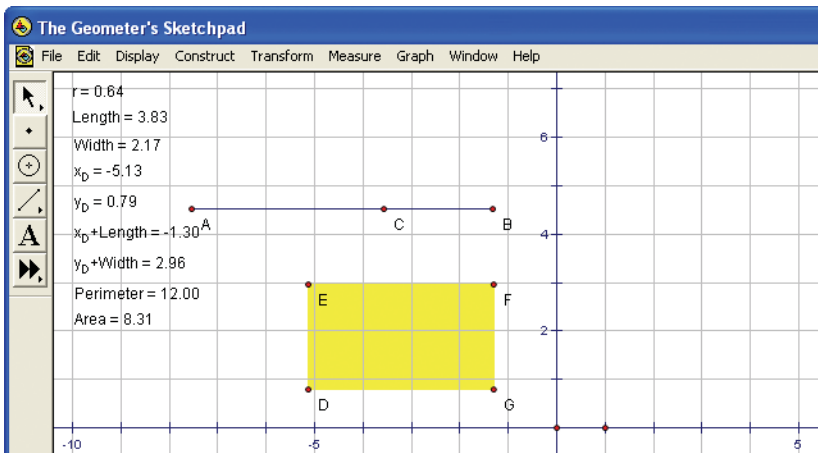
9. Select points D, E, F, and G, in that order. From the **Construct** menu, choose **Quadrilateral Interior**.

10. Move point C back and forth on the slider. Notice how the dimensions of the rectangle change. If the rectangle goes off your screen, drag the unit point to adjust the scale of your sketch.



11. Select **Length** and **Width**. From the **Measure** menu, choose **Calculate**. Enter the formula 2*(**Length** + **Width**). Change the label to **Perimeter**.

12. Select **Length** and **Width**. From the **Measure** menu, choose **Calculate**. Enter the formula **Length*Width**. Change the label to **Area**.



13. Verify that the perimeter remains constant as you move point C on the slider back and forth.

14. Use the slider to experiment with different dimensions for the rectangular play area with a perimeter of 12 m.

a) Describe the shapes that occur.

b) Which dimensions create the greatest area? Describe the shape of this play area.

15. **Reflect** Which shape would you choose for the play area? Give reasons for your choice.

16. Save your sketch for use in later investigations.

Investigate B

What is the relationship between the perimeters of rectangles with the same area?

Tools

- grid paper

Optional

- spreadsheet software
- computers

A rectangular pet exercise area is to have an area of 36 m^2.

1. a) Sketch all the rectangles that have whole-number dimensions and an area of 36 m^2.

b) Copy and complete the table or use a spreadsheet like the one shown.

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1				36
2				36
3				36

	A	B	C	D	E
1	Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
2	1	1	=36/B2	=2*B2+2*C2	36
3	2	=B2+1	=36/B3	=2*B3+2*C3	36
4					

2. a) What dimensions use the least amount of fencing?

b) What dimensions use the greatest amount of fencing?

3. **Reflect** Which shape would you choose for the pet exercise area? Give reasons for your choice.

Key Concepts

- Use the problem solving process to conduct an investigation.
  - Understand the problem so that you can investigate it properly. Is the perimeter fixed? Is the area fixed?
  - Choose a strategy for the investigation. Use manipulatives, such as toothpicks or geoboards and elastics, or use technology, such as a spreadsheet or *The Geometer's Sketchpad*®, whichever is appropriate for the investigation.
  - Record the results of each investigation so that you can refer back to them later.
  - When drawing conclusions from an investigation, always reflect on your answer. Ask yourself if you need to consider other factors.
- Rectangles with the same perimeter can have different dimensions and contain different areas.
- Rectangles with the same area can have different dimensions and different perimeters.

Communicate Your Understanding

C1 Describe how you could use grid paper to investigate the areas of rectangles with a perimeter of 40 units.

C2 Describe how you could use a geoboard to investigate the perimeters of rectangles with an area of 15 square units.

Practise

1. Explore the different rectangles that you can form with a perimeter of 24 units.

a) What are you to investigate?

b) Choose a strategy that you can carry out on grid paper. Record the areas of five different rectangles.

2. Explore the different rectangles that you can form with a perimeter of 20 units.

a) What are you to investigate?

b) Choose a strategy that you can carry out using toothpicks. Record the areas of three different rectangles.

3. Explore the different rectangles that you can form with an area of 12 square units.

a) What are you to investigate?

b) Choose a strategy that you can carry out using elastics on a geoboard. Record the perimeter of each rectangle.

Connect and Apply

4. You are designing a rectangular shed that has a floor area of 16 m^2. Using a geoboard, let the distance between the pegs represent 1 m.

a) With an elastic, construct different rectangles to represent the shed's floor. Record the dimensions of each rectangle you create in a table. Calculate the perimeter of each rectangle.

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1				16
2				16
3				16

b) Explain how the perimeter affects the cost of the shed.

c) Which shape would be the most economical for the shed? Why?

d) Is cost the only factor when choosing a shape for the shed? What other factors might you need to consider?

5. **Use Technology** Padma is making a vegetable garden in her yard. She wants to fence the garden to keep out small animals. She has 16 m of fencing. Use *The Geometer's Sketchpad*® to investigate the dimensions of the rectangular garden with the greatest area that Padma can enclose with this fencing.

6. Colin is enclosing a rectangular area for his dog with 32 m of fencing. Use a table or a spreadsheet to investigate the greatest area that Colin can enclose.

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	15	32	15
2	2	14	32	28

	A	B	C	D	E
1	Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
2	1	1	=16-B2	32	=B2*C2
3	2	=B2+1	=16-B3	32	=B3*C3
4					

Extend

7. What happens to the area when you change the shape of an enclosure? Suppose each toothpick represents a 1-m length of fence.

a) Use 36 toothpicks to build enclosures with the greatest area, using the following shapes:
- triangle
- rectangle
- hexagon
- circle

b) Find the area of each enclosure in part a).

c) Does the shape of the enclosure affect its area? Write a brief report on your findings.